Name: _____ Group _____

1) An individual who has automobile insurance from a certain company is randomly selected. Let X be the number of moving violations for which the individual was cited during the last 3 years. The mass of X is

x	0	1	2	3
$p_X(x)$	0.60	0.25	0.10	0.05

$\mathbb{E}(X) = 0.6$

a) Calculate the variance of the number of moving violations using Definition 12.5.

b) Calculate the variance of the number of moving violations using Remark 12.7.

c) Are the answers to parts a) and b) the same? Which method is easier?

2) An individual who has automobile insurance from a certain company is randomly selected. Let Z be the number of moving violations for which the individual was cited during the last 3 years. The mass of Z is

z	0	1	2	3
$p_Z(z)$	0.45	0.30	0.20	0.05

$\mathbb{E}(Z) = 0.85$ Var(Z) = 0.8275

The cost of insurance depends on the following function of accidents, g(z) = 400 + (100z- 15),

a) Calculate the variance of the cost of insurance by creating a new mass and then calculating the variance.

b) Calculate the variance of the cost of insurance using Remark 12.13.

c) Are the answers to parts a) and b) the same? Which method is easier?

3) An individual who has automobile insurance from a certain company is randomly selected. Let W and Y be the distributions of the number of moving violations for which individuals were cited during the last 3 years. . Assume that Y and W are independent. The masses of Y and W are

y	0	1	2	3
$p_Y(y)$	0.30	0.35	0.15	0.10

$\mathbb{E}(Y) = 0.95$

w	0	1	2	3
$p_W(w)$	0.15	0.45	0.25	0.15

$\mathbb{E}(W) = 1.4$, $Var(W) = 0.84$

b) Find Var(Y).

b) Find Var(3Y – 4W)?

c) Is Var(3Y + 4W) the same as for part b? Why?

d) Find Var(3Y).

e) Find Var(Y + Y + Y)?

f) Are the answers to parts d) and e) the same? Why?

4) At a restaurant that sells appetizers:
8% of the appetizers cost $1 each
20% of the appetizers cost $2 each.
32% of the appetizers cost $3 each.
40% of the appetizers cost $4 each.

An appetizer is chosen at random, and X is its price. Each appetizer has a 7% sales tax. So Y = 1.07X is the amount paid on the bill (in dollars).

a) Find the variance of Y.

b) What is the variance of a drink and an appetizer if all drinks are $2 (plus tax)?

c) What is the variance of 5 independent people who order appetizers (with drinks)?"

d) If W is the random variable in part a). Let Z = 5W. What is the variance of Z?

5) A box contains 10 disks of radii 1, 2, ..., 10, respectively. What is the expected value of the area of a disk selected at random from the box?

6) Prove that if P(X = c) = 1 for some c, then Var(X) = 0. Hint: Use the definition of variance.

7) Show Corollary 12.20 (without using Theorem 12.19) Hint: Use the definition of variance and rules for expected values. No explanation of steps is required.

a) $Var(bX) = b^2Var(X)$

b) Var(X + a) = Var(X) "

8) For a standardized random variable X

$$X^* = \frac{X - \mathbb{E}(X)}{\sqrt{Var(X)}}$$

a) Calculate the expected value of X*.

b) Calculate Var(X*)

9) A random sample of size n is taken from a very large lot of items in which 100p1% have exactly one defect and 100p2% have two or more defects, where $0 < p1 + p2 < 1$. An item with exactly one defect costs \$1 to repair, whereas an item with two or more defects costs \$3 to repair. Determine the expected cost of repairing the defective items in the sample. The final answer will be a function of p_1 and p_2.

10) This problem involves the scenario in problem 5 on Worksheet W6M and Problem 7 on Worksheet W7W. Consider some 4-sided dice. Roll two of these dice. Let X denote the minimum of the two values that appear, and let Y denote the maximum of the two values that appear.

a) Find Var(X)

b) Find Var(Y)

c) BONUS: These two numbers are the same. Can you think of a reason this has to happen or is this a coincidence?